

Notes - HW 17: Interest

Module 4: Managing Money and Interest

Unit B The Power of Compounding

Objectives:

1. Use the simple interest formula
2. Use the compound interest formula
3. Solve applications for interest problems

If you have the chance to invest money, this section will look at the differences between simple and compound interest... **Interest** is the money that one person pays another person when they borrow money. **Interest** is also the money that one person earns from another person when they loan money. Savers **EARN interest**; borrowers **PAY interest**. Remember... that loans require repayment of the principal AND interest. If we only repay the interest we will never pay the loan off!

Suppose you are going to deposit money in the bank. The amount that you deposit is called the **principal** and the amount of interest you earn is determined by the principal, the interest rate, and the time.

Simple interest is calculated by the formula below and is for one time interest calculations.

$I = Prt$ where I is the interest earned, P is the principal invested, r is the annual interest rate, and t is the time in years.

Example 1

Question: If you deposit \$1000 in a bank account paying 5.5% annual interest, how much interest will the deposit earn in 3 years if the bank computes the interest using simple interest?

Solution:

Notice that the principal $P = 1000$, $r = 0.055$, and $t = 3$.

Using the simple interest formula, we find:

$$I = 1000(0.055)(3) = 165$$

Now we see that the interest earned over 3 years is \$165. (And at the end of 3 years, we would have a total of $1000 + 165 = \$1165$.)

Not only should we calculate interest earned but the **future value** is important as well. Essentially, we start with present value and add the interest earned to create future value.

$A = \text{principal} + \text{interest} = \text{future value}$

$$A = P + I$$

$$A = P + Prt$$

$$A = P(1 + rt)$$

Example 2

Question: Assume that you plan to save \$2,500 to take a white-water rafting trip in two years. Your bank offers a certificate of deposit (CD) that pays 4% annual interest computed using simple interest. How much should you invest now?

Solution: Notice that the principal is unknown, $r = 0.04$, $t = 2$ and the future value A is \$2500. Start by writing the formula and then substituting the correct values. We will eventually solve for P and this value should be less than \$2500.

$$\begin{aligned}A &= P(1 + rt) \\2500 &= P(1 + 0.04 \times 2) \\2500 &= P(1 + 0.08) \\2500 &= P(1.08) \\\frac{2500}{1.08} &= P \\P &= \$2314.81\end{aligned}$$

So, we will need \$2314.81 to guarantee \$2500 in two years.

Keep in mind, simple interest is not our only option... **IF** we can compound interest earned on top of interest on top of interest)..., we would have an **even larger future value**. Banks will calculate interest on a daily, monthly, quarterly, and annual (APR) basis and our investments will grow at a much faster rate!! Interest that is recalculated at regular time periods is called **compound interest**...

Here is what happens to an initial invest of \$2000 with an interest rate (APR) of 10%, compounded annually for 3 years:

Year	Principal (beginning of year) P	Future Value (end of year) $P(1 + rt) = P(1 + 0.10 \times 1) = P(1.10)$
1	\$2000	$2000(1.10) = \$2200$
2	\$2200	$2200(1.10) = \$2420$
3	\$2420	$2420(1.10) = \$2662$

Notice that the interest is calculated at the end of each year and the principal at the beginning of the next year is higher than the previous year.

This idea leads us to the **compound interest** formula:

$$A = P \left(1 + \frac{r}{m} \right)^{mt}$$

where A is the future value, r is the annual interest rate (APR), m is the number of compoundings in one year, and t is the time in years.

Note:

$m = 1$ (compounded annually or APR)

$m = 4$ (compounded quarterly)

$m = 12$ (compounded monthly)

$m = 365$ (compounded daily)

Example 3

Question: You have seen a home fitness center on sale for \$3500 and what really makes the deal attractive is that there is no money down and no payments for 6 months. Realize that although you do not have to make any payments, the dealer is not loaning you the money for 6 months for nothing. You have borrowed \$3500 and in 6 months, your payments will be based upon that fact. Assuming your dealer is charging an annual interest rate of 12% compounded monthly, what interest will accumulate on your purchase over the next 6 months?

Solution: For this situation, $P = 3500$, $r = 0.12$, $m = 12$ (compounded monthly) for six months (half a year):

$$A = P \left(1 + \frac{r}{m} \right)^{mt}$$

$$A = 3500 \left(1 + \frac{0.12}{12} \right)^{12 \times \frac{1}{2}}$$

$$A = 3500(1 + 0.01)^6$$

$$A = 3500(1.01)^6$$

$$A = \$3715.33$$

This means that we if borrowed \$3500 and after 6 months, we OWE \$3715.33 or $3715.33 - 3500 = \$217.33$ in interest. **IF we want to avoid the interest altogether, we need to pay the \$3500 off BEFORE the end of 6 months.

Example 4

Question: Use the compound interest formula to determine the accumulated balance after the stated period: \$4000 invested at an APR (r) of 6% for 5 years... $m = 1$ for this problem for annual compounding.

$$A = P \left(1 + \frac{r}{m} \right)^{mt}$$

$$A = 4000 \left(1 + \frac{0.06}{1} \right)^{1 \times 5}$$

$$A = 4000(1 + 0.06)^5$$

$$A = 4000(1.06)^5$$

$$A = \$5352.90$$

Example 5

Question: Use the appropriate compound interest formula to compute the balance in the account after the stated period of time: \$9,000 is invested for 4 years with an APR (r) of 7% and quarterly compounding ($m = 4$).

$$A = P \left(1 + \frac{r}{m} \right)^{mt}$$

$$A = 9000 \left(1 + \frac{0.07}{4} \right)^{4 \times 4}$$

$$A = 9000(1 + 0.0175)^{16}$$

$$A = 9000(1.00175)^{16}$$

$$A = \$11,879.36$$

Example 6

Question: Use the appropriate compound interest formula to compute the balance in the account after the stated period of time: \$11,000 is invested for 10 years with an APR (r) of 7% and monthly compounding (m = 12).

$$A = P \left(1 + \frac{r}{m} \right)^{mt}$$

$$A = 11000 \left(1 + \frac{0.07}{12} \right)^{12 \cdot 10}$$

$$A = 11000 (1 + 0.0058333...)^{120}$$

$$A = 11000 (1.0058333...)^{120}$$

$$A = \$22,106.28$$

****Note...** do NOT round until the final answer and that way you get the most precise answer! ☺

Example 7

Question: Upon the birth of a child, a parent wants to make a deposit into a tax-free account to use later for the child's college education. Assume that the account has an annual interest rate of 4.8% and that the compounding is done **quarterly**. How much must the parent deposit now so that the child will have \$60,000 at age 18?

Solution: For this situation, A = 60,000, r = 0.048, m = 4 (**compounded quarterly**) for 18 years. We will eventually solve for P and this answer should be less than \$60,000.

$$A = P \left(1 + \frac{r}{m} \right)^{mt}$$

$$60000 = P \left(1 + \frac{0.048}{4} \right)^{4 \cdot 18}$$

$$60000 = P (1 + 0.012)^{72}$$

$$60000 = P (1.012)^{72}$$

$$60000 = P (2.360461386)$$

$$\frac{60000}{2.360461386} = P$$

$$P = \$25,418.76$$

So the parents need \$25,418.76 and that would grow to \$60,000 in 18 years. WOW!!!

Example 8

Question: Last quarter, sales of women's sandals in a retail store were \$12,650. The goal for this quarter is to increase sales by 12%. What is the goal for this quarter in dollars?

Solution: Here is one way to solve this problem...

$$\frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{percent of increase}}{100}$$

$$\frac{x}{12650} = \frac{12}{100}$$

$$100x = 151800$$

$$x = \frac{151800}{100} = \$1,518$$

$$\begin{aligned}\text{New value} &= \text{original amount} + \text{increase amount} \\ &= 12650 + 1518 \\ &= \$14,168 \text{ (goal for this quarter)}\end{aligned}$$